PCAC and shadowing of low energy neutrinos

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The Adler relation between reactions initiated by neutrinos and pions is easy to misinterpret as a manifestation of the pion pole dominance. An axial current, however, cannot fluctuate into a pion, but only to heavy axial-vector states, since the lepton current is transverse. This is the miracle of the PCAC hypothesis which dictates a specific conspiracy between the heavy fluctuations, so that all together they mock the pion pole. Indeed, the observed Q^2 dependence of the axial form factor is controlled by the effective mass $m_A \sim 1\,\mathrm{GeV}$, rather than the pion mass. On the contrary, the onset of nuclear shadowing is governed by the small pion mass, rather than by the large axial mass scale. This is in variance with the conventional wisdom which equates the fluctuation lifetime and the coherence time. For the case of axial current they are different by almost two orders of magnitude. As a result, neutrino interactions are shadowed at very low energies of few hundred MeV, while energy of about 10 GeV is needed to access nuclear shadowing for the vector current. On the contrary to naive expectations, nuclear absorption enhances, rather than suppresses the cross section of coherent neutrino-production of pions which is the strongest channel (half of the total cross section) in the black disc limit.

1. Nuclear shadowing of the axial current

It is known that even weakly interacting particles are shadowed by nuclei, i.e. the interaction cross section per a bound nucleon is less than on a free one. Shadowing results from competition of different bound nucleons in taking part in the interaction. This may produce a sizeable effect only provided that the interaction cross section is sufficiently large. Apparently, this is not the case for electromagnetic or weak interactions, and one may wonder how shadowing happens. The answer has been known for decades, namely, a weakly interacting particle may develop a hadronic strongly interacting fluctuation. Of course the probability of such a fluctuation is tiny, but if its lifetime is longer than the time of propagation through the nucleus, then once produced this fluctuation experiences nuclear shadowing which is as strong as in hadronic interactions.

The fluctuation lifetime is controlled by the uncertainty principle and Lorentz time dilation,

$$t_{fluct} = \frac{2\,E}{M_{eff}^2}\ , \tag{1}$$

where E and M_{eff} are the energy and effective mass of the fluctuation. This is also frequently called coherence time, t_c, as it defines the maximal time interval between two interactions with amplitudes which have a small phase shift, i.e. are coherent. We will show, however, that this usual equivalence of the two time scales is not correct for the axial current. Namely, the coherence time in this case is almost two orders of magnitude longer than the one given by Eq. (1).

1.1. PCAC and hadronic properties of neutrino

The electric charge is not renormalized by the strong interactions due to vector current conservation, $q_\mu V_\mu = 0$. One may think it is almost trivial, since $q_\mu\,\bar{p}\gamma_\mu n = m_n - m_p \approx 0$.

Data show that the axial charge is also hardly changed by the strong interactions, pointing at a partial conservation of axial current (PCAC). This is a very nontrivial phenomenon, since $q_\mu\,\bar{p}\gamma_5\gamma_\mu n = m_n + m_p$ is a big quantity. It can be, however, compensated by the effective pseudo-scalar term, $g_p\,q_\mu\,\bar{p}\gamma_5\,n$, generated in the axial current by the strong interactions. Such a compensation is possible only if the pseudo-scalar coupling has a pole, $g_p(q^2) \propto 1/q^2$, corresponding to a massless pseudo-scalar, Goldstone

meson, which must exist to provide PCAC. This leads to the Goldberger-Treiman relation between the couplings $g_{NN\pi}$ and f_π and to the PCAC relation for the axial current,

$$\partial_\mu A_\mu = f_\pi\, m_\pi^2\, \phi_\pi\ . \tag{2}$$

The PCAC leads to a relation between the cross sections of interaction of neutrino and pion, named after Stephen Adler [1],

$$\left.\frac{d\sigma(\nu T\to lF)}{dQ^2\,d\nu}\right|_{Q^2=0} = \frac{G^2}{2\pi^2}\, f_\pi^2 \left(\frac{1}{\nu}-\frac{1}{E}\right) \sigma(\pi T\to F)\ . \tag{3}$$

Here F is the hadronic finals state produced on target T either by the neutrino (left-hand side), or by the pion (right-hand side); E is the energy of the neutrino; ν is the transferred energy.

The structure of this relation is very similar to one suggested for the vector current by the vector dominance model (VDM),

$$\left.\frac{d\sigma(\nu T\to lF)}{dQ^2\,d\nu}\right|_{Q^2=0} = \frac{G^2}{4\pi^2}\, f_\rho^2\, \frac{|\vec{q}|}{E_\nu^2}\, \frac{Q^2}{(Q^2+m_\rho^2)^2} \times \frac{1}{1-\varepsilon}\Big[\sigma_T(\rho T\to F)+\varepsilon\,\sigma_L(\rho T\to F)\Big]\ , \tag{4}$$

where ε is the ρ polarization which might be transverse (T) or longitudinal (L).

In both cases of axial, Eq. (3), and vector, Eq. (4), currents the neutrino cross section is proportional to the hadronic, π, or ρ, cross sections, which are subject to a substantial shadowing if the target T is a nucleus. The intuitive light-cone interpretation of the VDM considers different Fock components of the vector current assuming that vector mesons, in particular ρ, are the dominant ones. If the lifetime of these fluctuations, Eq. (1) is sufficiently long, shadowing effects are at work[1]

This is why it is tempting to interpret the Adler relation Eq. (3) as a manifestation of pion dominance, i.e. a fluctuation of the axial current to a pion which interacts with the target. Such a fluctuation, however, is forbidden. Indeed, the Lorentz structure of the hadronic current in this case would be,

$$A_\mu = f_\pi\, \frac{q_\mu}{Q^2+m_\pi^2}\, A(\pi T\to F)\ . \tag{5}$$

[1]Note that in this case $t_{fluct}=t_c$.

The factor q_μ acting on the transverse lepton current gives zero, or a tiny lepton mass, $q_\mu\, \bar{l}\,\gamma_5\gamma_\mu\,\nu = m_l$.

Thus, a neutrino can produce only heavy axial fluctuations, like a_1-meson, $\rho\pi$ pair, etc. This looks like a miracle that all those states add up and act like a pion, and this fine tuning must be independent of the target. Nevertheless, this is what the PCAC relation is about. Unfortunately, the details of this phenomenon are beyond our knowledge of hadronic dynamics, and we should treat it as a hypothesis aimed at explanation of the observation of the nearly conserved axial coupling.

To be convinced that PCAC is indeed provided by heavy hadronic fluctuations, rather than a pion, one can look at the Q^2-dependence of the cross section. It is given by the fluctuation propagator, therefore the width of the Q^2 distribution gives the effective fluctuation mass. Indeed, data depicted in Fig. 1 clearly demonstrate the following features:

- There exists a longitudinal part of the cross section which does not vanish at $Q^2\to 0$;
- Its magnitude well agrees with the Adler relation;
- The axial mass parameter controlling the Q^2 dependence, is large, $m_A^2 \sim 1\,\mathrm{GeV}^2$, two orders of magnitude larger than the pion mass squared.

Another unusual property of the axial current which is worth mentioning: there is no vector dominance in this case, though it is tempting to assume that like for the vector current the contribution of the lowest axial vector meson dominates. This assumption has led, however, to a problem called Piketty-Stodolsky puzzle [3]. Namely, the cross section of the off-diagonal diffractive process $\pi N\to a_1 N$ is so small that provides only a few percent of the observed cross section of neutrino-production of pions. The main contribution comes from the $\rho\pi$ cut [4,2].

1.2. Shadowing of low energy neutrinos

As far as the effective mass of a typical hadronic fluctuation of a neutrino is as large as $1\,\mathrm{GeV}$, quite a high energy, $\nu \gtrsim 10\,\mathrm{GeV}$, is needed, to make the fluctuation lifetime Eq. (1) comparable with the radii of heavy nuclei. Thus, one may jump to a conclusion that there should be no shadowing at low energies.

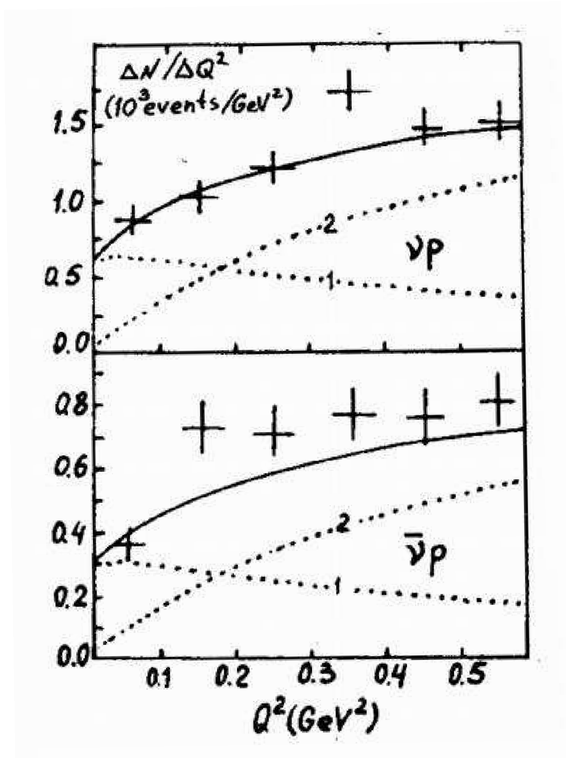

Figure 1. Q^2 dependence of the total νp and $\bar{\nu} p$ cross sections. The dotted curves, **1** is an extrapolation to nonzero Q^2 of the Adler relation; **2** is calculation employed VDM for the vector and axial current. Solid curve show the sum of the two contributions. Data are taken from [2].

However, this conclusion is not correct. It is based on the usual wisdom that the fluctuation lifetime and the coherence time are equivalent quantities, what is not true in this case. Indeed, for elastic neutrinoproduction of pions, $\nu p \to l \pi N$, the longitudinal momentum transfer, $q_L = (m_\pi^2 + Q^2)/2\nu$, is rather small at $Q^2 \lesssim m_\pi^2$, i.e. the coherence time, $t_c = 1/q_L$ is very long even at low energy of few hundred MeV. This is actually what matters for shadowing. As for the fluctuation lifetime, it is indeed much shorter.

This is a result of the nontrivial origin of PCAC. Impossibility to have a pion in intermediate state leads to the dominance of off-diagonal processes, like $\nu \to \mu a_1$ and $a_1 N \to \pi N$. Same happens for the vector current, if one considers, for example, ρ photoproduction via intermediate excitation ρ': $\gamma \to \rho'$ and $\rho' N \to \rho N$. Such a off-diagonal contribution is negligibly small for the vector current, but is a dominant one for neutrinos. Only for diagonal transitions the fluctuation lifetime and the coherence time are equal, $t_{fluct} = t_c$. For neutrino interactions the former controls the Q^2 dependence of the cross section, while the latter governs shadowing.

The total neutrino-nucleus cross section was calculated in [5,6] taking into account the phase shifts between different points of interaction (effects of coherence). The calculations are performed using the Glauber approximation and also including the Gribov's inelastic corrections (important only at high energies). The results for neon are depicted in Fig. 2 by solid curves as function of energy for different Q^2. As

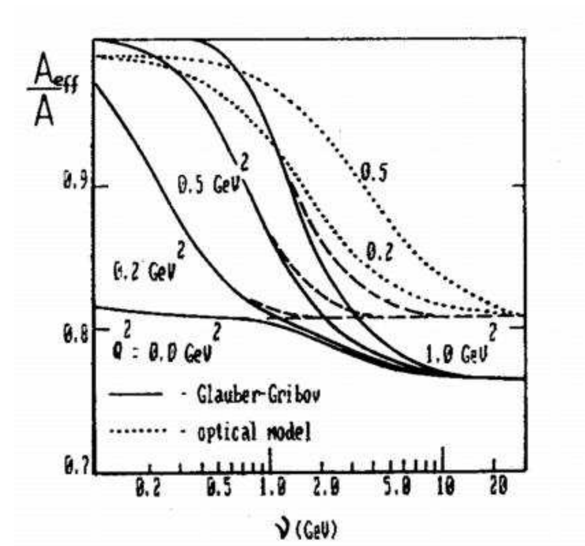

Figure 2. Energy dependence of the ratio of total neutrino cross sections on neon and nucleon at different Q^2 [5,6]. Dashed curves correspond to the Glauber approximation. Solid curves are corrected for the Gribov's inelastic shadowing. Dotted curves show the results of the Bell's optical model [7].

was anticipated, a rather strong shadowing occurs at small Q^2 in the low energy range of hundreds MeV. This is an outstanding property of the axial current.

The calculations are in a good agreement with available data from the BEBS experiment at CERN [8] depicted in Fig. 3. Although with a rather poor statistics the data confirm an early onset of nuclear

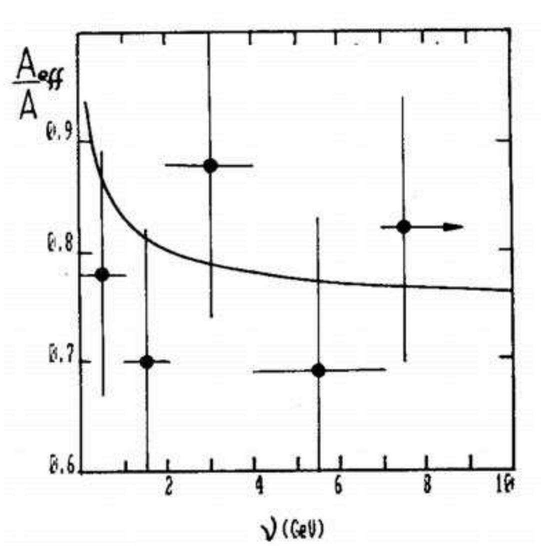

Figure 3. The neon to proton ratio of the total neutrino cross sections, calculated in [5,6] for $x < 0.2$ and $Q^2 < 0.2\,\mathrm{GeV}^2$. The data are from [8].

shadowing at energies below 1 GeV.

2. Diffractive neutrino-production of pions

2.1. Pion production on free nucleons

The differential cross section of this reaction on a nucleon target is given by the Adler relation,

$$\left.\frac{d\sigma(\nu N \to \mu\pi F)}{dQ^2\,d\nu\,d^2k_T}\right|_{Q^2=0} = \frac{G^2}{2\pi^2}\,f_\pi^2\left(\frac{1}{\nu}-\frac{1}{E}\right)\frac{d\sigma_{el}^{\pi N}}{d^2k_T}\,. \quad (6)$$

This expression can be extrapolated to nonzero values of Q^2 using a form factor parametrized usually in a pole form,

$$F_N(Q^2) = \frac{1}{1+Q^2/m_A^2} \quad (7)$$

which well fits data with m_A close to the mass of a_1 meson. This fact is treated sometimes as an evidence for axial-vector meson dominance, but as was mentioned, that would lead to the Piketti-Stodolsky puzzle. It turns out, however, that the $\rho\pi$ cut provides the dominant contribution to the cross section, leading to a pole-like Q^2 dependence. Indeed, the form-factor calculated using the Deck model [9] reads [4],

$$F_N^{\rho\pi}(Q^2) = \frac{(m_\rho^2+m_\pi^2)^2}{m_\pi^2+Q^2}\,\ln\left[1+\frac{m_\pi^2+Q^2}{(m_\rho+m_\pi)^2}\right] \approx \frac{1}{1+Q^2/m_A^2} \quad (8)$$

where $m_A^2 = 2(m_\rho^2+m_\pi^2)$ is indeed very close to the a_1 mass. Thus, the cut contribution mocks the a_1 pole.

2.2. Coherent production off nuclei

It is the commonly accepted terminology to call *coherent* a process which leaves the nuclear target intact in its ground state. The cross section for coherent production of pions reads [4,10],

$$\frac{d\sigma(\nu A \to \mu\pi A)}{d\nu\,dQ^2\,dk_T^2} = \frac{G^2 f_\pi^2}{2\pi^2}\left(\frac{1}{\nu}-\frac{1}{E}\right)F_N^2(Q^2)\,\Phi_{coh}(k_T,k_L)\,, \quad (9)$$

where

$$\Phi_{coh}(k_T,k_L) = \frac{|\tilde\sigma_{tot}^{\pi N}|^2}{16\pi}\left|\int d^2b\,e^{i\vec b\cdot\vec k_T}\int_{-\infty}^{\infty}dz\,e^{izk_L} \times \rho_A(b,z)\,\exp\left[-\frac{\tilde\sigma_{tot}^{\pi N}}{2}\,T_z(b,z)\right]\right|^2. \quad (10)$$

Here the integration is taken over impact parameter $\vec b$ and longitudinal coordinate z; $\vec k_T$ and $k_L \approx (Q^2+m_\pi^2)/2\nu$ are the transverse and longitudinal momentum transfers; ρ_A is the nuclear density and $T_z(b,z) = \int_z^\infty dz\,\rho_A(b,z)$ is the nuclear thickness; $\tilde\sigma_{tot}^{\pi N} = \sigma_{tot}^{\pi N}\cdot(1-i\alpha_{\pi N})$, where $\alpha_{\pi N}$ is the real to imaginary part ratio for the forward elastic πN amplitude.

The result of calculations [4] for the cross section Eq. (9) for neon target is depicted in Fig. 4 as function of energy. Comparison with data demonstrates good agreement.

The phase factor in (10) oscillates at low energy and suppresses the coherent pion production, as one can see in the figure. On the contrary, at high energies, in the limit of $k_L R_A \ll 1$ the phase factor in (10) can be neglected and the differential cross section Eq. (9) of neutrinoproduction of pions is proportional to the cross section of elastic pion-nucleus collisions, i.e. Eqs. (9)-(10) become equivalent to the Adler relation. For very heavy nuclei (black disk limit) the elastic $\pi - A$ cross section reaches the maximum, πR_A^2.

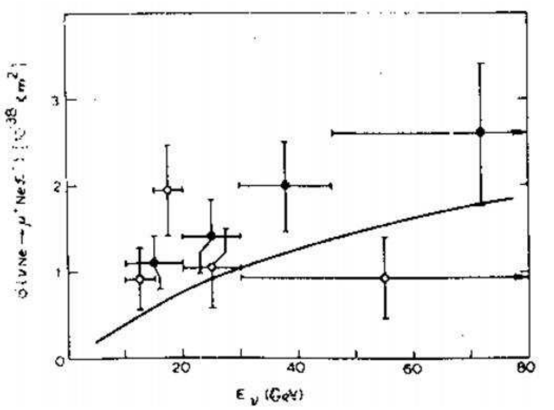

Figure 4. Energy dependence of the cross section of coherent neutrino-production of pions on neon. Data are from [11,12].

Note that the popular Rein-Sehgal model [13] predicts zero cross section in this limit contradicting quantum mechanics and the Adler relation.

2.3. Incoherent production

Neutrino can produce diffractively a pion on a bound nucleon, $\nu N \to \mu\pi N$, and knock the nucleon out of the Fermi surface. In this case the bound nucleons act incoherently, and the nucleus breaks up into fragments. Such a process has a weaker A-dependence ($A^{1/3}$ compared to $A^{2/3}$ for coherent production), but is not suppressed by the nuclear form factor at low energy, where it turns out to be the dominant contribution to pion production.

The cross section of incoherent (quasielastic) neutrinoproduction of pions reads [4,14,10],

$$\frac{d\sigma(\nu A \to \mu\pi A^*)}{d\nu\, dQ^2\, dk_T^2}$$
$$= \frac{G^2 f_\pi^2}{\pi^2}\,\frac{EE' - Q^2/4}{2E^2|\vec{q}\,|}\,F_N^2(Q^2)\,\Phi_{inc}(k_T,k_L)\,, \quad (11)$$

where $E' = E - \nu$, and

$$\Phi_{inc}(k_T,k_L) = \frac{|\tilde{\sigma}_{tot}^{\pi N}|^2}{16\pi}\,\exp\left(-B_{\pi N}\,k_T^2\right)$$
$$\times \int d^2b\,\left\{\frac{1}{\tilde{\sigma}_{in}^{\pi N}}\left[1 - \exp\left(-\tilde{\sigma}_{in}^{\pi N}\,T_A(b)\right)\right]\right.$$
$$+ \frac{\tilde{\sigma}_{tot}^{\pi N}\left(\tilde{\sigma}_{in}^{\pi N} - \tilde{\sigma}_{el}^{\pi N}\right)}{2\tilde{\sigma}_{el}^{\pi N}}\int\limits_{-\infty}^{\infty} dz_1\,\rho_A(b,z_1)\,e^{-ik_L z_1}$$
$$\times \exp\left[-\frac{1}{2}\,\tilde{\sigma}_{tot}^{\pi N}\,T_z(b,z_1)\right]\int\limits_{z_1}^{\infty} dz_2\,\rho_A(b,z_2)\,e^{ik_L z_2}$$
$$\times \left.\exp\left[-\frac{1}{2}\left(\tilde{\sigma}_{in}^{\pi N} - \tilde{\sigma}_{el}^{\pi N}\right)T_z(b,z_2)\right]\right\}. \quad (12)$$

There are two terms in this expression, the fist one corresponds to the low energy limit when the oscillations terminate the second term. This first term has pure classical form with no interferences. In the high energy limit, when $k_L \ll 1/R_A$, the whole expression takes the form of the cross section for quasielastic pion-nucleus scattering.

Example of numerical calculations is presented in Fig. 5 where both coherent and incoherent cross sections are compared with data [11] for neon.

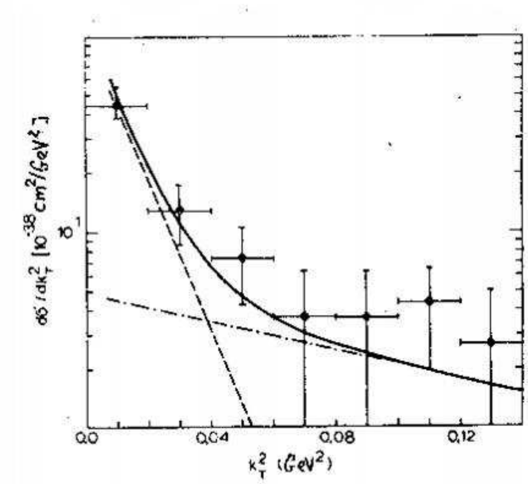

Figure 5. k_T^2 dependence of the cross sections of neutrinoproduction of pions on neon, coherent (dashed) and incoherent (dashed-dotted). The sum of the cross section is shown by solid curve. The data are from [11].

3. Conclusions

To summarize, we highlight the main observations of this talk.

- PCAC is a hypothesis suggested by the observed small effect of renormalization of the axial charge. Although this hypothesis has

well passed the low energy tests, like the Goldberger-Treiman relation, further tests are much encouraged. In particular, neutrino interactions providing an intensive source of axial current should be used for this purpose [2].

- One may mistreat PCAC and the Adler relation as a manifestation of pion dominance for the axial current. However, neutrino cannot emit a pion fluctuation because of transversity of the lepton current.

- The dispersion relation for the axial current is dominated by heavy states with mass of the order of 1 GeV. Probably the most nontrivial and intriguing property of PCAC is that all those heavy states conspire in a way that they mock the pion pole contribution. The observed Q^2 dependence of the cross section indeed confirms that the effective mass is $m_A \sim 1\,\mathrm{GeV}$.

- The axial current exhibits neither pion dominance, nor axial-vector meson (a_1) dominance. The latter is due to the strong suppression of diffractive $\pi \to a_1$ transitions observed in data. The important contribution to the dispersion relation for the axial current is the $\rho\pi$ cut. The corresponding axial form-factor has a form imitating the a_1 pole contribution.

- In spite of the dominance of heavy fluctuations, the onset of nuclear shadowing for neutrinos is controlled by the pion mass. This seems to contradict the intuition based on the conventional wisdom suggesting that the coherence time and fluctuation lifetime are the same things. However, in the case of off-diagonal processes, like one under discussion, this is not true. The lifetime of fluctuations is much shorter than the coherence time.

- The coherence time for the axial current at small Q^2 turns out to be about two orders of magnitude shorter than for the vector current. This nontrivial observation leads to an onset of nuclear shadowing for neutrinos at extremely low energies, hundreds of MeV.

- The strong channel of neutrino interaction, coherent pion production is enhances, rather than suppressed by nuclear absorption. The heavier and less transparent is the nucleus, the more pions are produced coherently. In the black disk limit (everything is absorbed) this is the strongest channel of neutrino-nucleus interaction.

- Although coherent neutrino-production is the dominant source of pions at high energies, the incoherent process takes over at low energies, where the coherent process is substantially suppressed by the nuclear form factor.

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